

# Skills from previous math classes that you need to self-review for Math 43

## From Algebra:

Equations of lines (slope-point form)  
Graphs of linear equations  
Quadratic functions  
    Graphing  
    Factoring  
    Quadratic formula  
Rational expressions  
    Add / subtract  
Exponentials and logarithms  
    Simplifying expressions  
    Solving equations

## From Geometry:

Areas  
    Triangles / parallelograms

## From Trigonometry:

Sine / cosine / tangent of special angles on unit circle  
Inverse sine / cosine / tangent of special values  
Pythagorean / reciprocal / quotient / negative angle / co-function identities  
Double angle / sum & difference of angles identities  
Trigonometric equations

## From Precalculus:

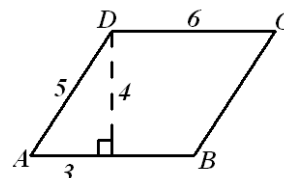
Graphs of basic functions (domain, range, intercepts, asymptotes, long run behavior)  
    Power  $y = x^n$  ( $n$  could be positive or negative, even or odd or reciprocal of integer)  
    Exponential  $y = b^x$  ( $b$  could be greater than or less than 1)  
    Logarithmic  $y = \log_b x$  ( $b$  could be greater than or less than 1)  
    Trigonometric  $y = \sin x$ ,  $y = \cos x$ ,  $y = \tan x$ ,  $y = \csc x$ ,  $y = \sec x$  or  $y = \cot x$   
    Inverse trigonometric  $y = \sin^{-1} x$ ,  $y = \cos^{-1} x$  or  $y = \tan^{-1} x$   
Symmetry of functions & graphs (relationship between algebraic & graphical symmetry)  
    Even / odd

**You must be able to solve these**  
**using neither your calculator nor any external aid**  
**All answers must be completely simplified**

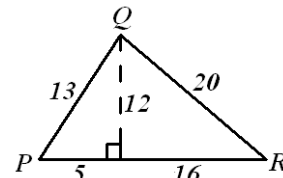
- [1] Find the **slope-point** form of the equation of the line through the points  $(-1, -3)$  and  $(-6, 4)$ .
- [2] Sketch the graph of the equation  $2x - 3y = -12$  by finding the  $x$  - and  $y$  - intercepts.
- [3] Sketch  $f(x) = x^2 - 6x - 16$  by finding the  $x$  - and  $y$  - intercepts and the vertex (without any additional points).
- [4] Solve  $3x^2 - 2x = 9$ .

- [5] Add and simplify  $\frac{x-7}{x^2-4x+3} + \frac{x+7}{x^2-x-6}$ .

- [6] Find the area of the parallelogram  $ABCD$  on the right.



- [7] Find the area of the triangle  $PQR$  on the right.



- [8] Fill in the following table with all **function** values (in radians) that have exact values. (Some entries have values which can only be found using a calculator. Mark those as "NEED CALC".) Also, identify the entries which do not exist (ie. have no function value).

$x =$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{2}}{2}$	$-\sqrt{3}$	$-\frac{1}{2}$	$-1$	$0$	$1$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{2}$
$\tan^{-1} x =$													
$\cos^{-1} x =$													
$\sin^{-1} x =$													

- [9] Let  $\theta = \frac{\pi}{6}$ .
- [a] Find an angle with positive measure that is co-terminal with  $\theta$ .
- [b] Find an angle with negative measure that is co-terminal with  $\theta$ .
- [c] Find 3 angles between  $0$  and  $2\pi$  that have  $\theta$  as their reference angle.

- [10] State the following trigonometric identities.

- [a] the 3 Pythagorean identities that involve the 6 trigonometric functions
- [b] the co-function identities for each of the 6 trigonometric functions
- [c] the double angle identities for  $\cos 2x$  (3 versions) and  $\sin 2x$

- [11] Simplify  $\sin(x - \pi)$ .

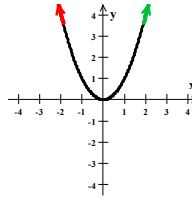
- [12] Simplify  $\cos(2\pi - x)$ .

- [13] Find all solutions of  $1 + 2\cos x = 0$ , where  $0 \leq x \leq 2\pi$ .

- [14] Find all solutions of  $\sin 2x = -\frac{\sqrt{3}}{2}$ .
- [15] Sketch the general shape and position of the following graphs. Do not worry about specific  $x$  - and  $y$  - coordinates.
- |                   |                   |                   |
|-------------------|-------------------|-------------------|
| $y = x^5$         | $y = x^{-4}$      | $y = \sqrt{x}$    |
| $y = e^x$         | $y = 0.5^x$       | $y = \ln x$       |
| $y = \cos x$      | $y = \tan x$      | $y = \csc x$      |
| $y = \sin^{-1} x$ | $y = \cos^{-1} x$ | $y = \tan^{-1} x$ |
- [16] Determine algebraically if  $f(x) = x\sqrt{1+x^2}$  is symmetric about the  $y$  - axis, about the origin or neither.
- [17] Determine algebraically if  $f(x) = \sin x - \cos x$  is even, odd or neither.
- [18] Fill in the blanks regarding the long run behavior of the following functions by referring to each function's graph. The answers should be  $\infty$ ,  $-\infty$ , a number, or DNE if the function does not tend towards a single direction, or does not exist as  $x$  gets large in size.

eg. As  $x \rightarrow \infty$ ,  $x^2 \rightarrow \underline{\infty}$

As  $x \rightarrow -\infty$ ,  $x^2 \rightarrow \underline{\infty}$



As  $x \rightarrow \infty$ ,  $e^x \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $e^x \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow \infty$ ,  $e^{-x} \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $e^{-x} \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow \infty$ ,  $x^5 \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $x^5 \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow \infty$ ,  $x^8 \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $x^8 \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow \infty$ ,  $x^{-6} \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $x^{-6} \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow \infty$ ,  $x^{-3} \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $x^{-3} \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow \infty$ ,  $\sin x \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $\sin x \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow \infty$ ,  $\ln x \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $\ln x \rightarrow \underline{\hspace{2cm}}$

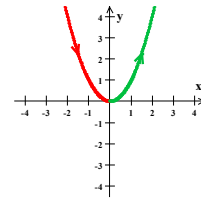
As  $x \rightarrow \infty$ ,  $\tan^{-1} x \rightarrow \underline{\hspace{2cm}}$

As  $x \rightarrow -\infty$ ,  $\tan^{-1} x \rightarrow \underline{\hspace{2cm}}$

- [19] Fill in the blanks regarding the intervals of increasing/decreasing behavior of the following functions by referring to each function's graph. Some functions will have multiple intervals over which they are increasing (or decreasing), or no intervals over which they are increasing (or decreasing).

eg. **Increasing:** As  $x$  goes from  $\underline{0}$  to  $\underline{\infty}$ ,  $x^2$  increases from  $\underline{0}$  to  $\underline{\infty}$

**Decreasing:** As  $x$  goes from  $\underline{-\infty}$  to  $\underline{0}$ ,  $x^2$  decreases from  $\underline{\infty}$  to  $\underline{0}$



As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $e^x$  increases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $e^x$  decreases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $-x^3$  increases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $-x^3$  decreases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $e^{-x}$  increases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $e^{-x}$  decreases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $\frac{1}{x^2}$  increases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

As  $x$  goes from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ ,  $\frac{1}{x^2}$  decreases from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .

[20] Simplify the following expressions if possible.

$$e^x + e^y =$$

$$\ln x + \ln y =$$

$$\ln(x + y) =$$

$$e^x - e^y =$$

$$\ln x - \ln y =$$

$$\ln(x - y) =$$

$$e^x e^y =$$

$$(\ln x)(\ln y) =$$

$$\frac{e^x}{e^y} =$$

$$\frac{\ln x}{\ln y} =$$

$$(e^x)^y =$$

$$(\ln x)^y =$$

$$\ln x^y =$$

$$e^{2 \ln x} =$$

$$e^{-\ln 2x} =$$

[21] Find all real solutions of the following equations.

$$[a] \quad 9 - 2e^{3x+1} = 4$$

$$[b] \quad 7 \ln(1 - 6x) + 5 = 8$$

$$[c] \quad e^{2x} - 2e^x = 8$$

**HINT:**

The questions above involve material from the following classes.

Geometry: [6], [7]

Algebra: Math 114 [1]-[5]

Precalculus I: Math 41 [15]-[21]

Trigonometry: Math 42 [8]-[15], [17]-[18]

**NOTE:**

There is no solution key for this prerequisite package since it only involves material that you have learned before.

You are encouraged to work together with your classmates, and to consult your old textbooks and notes.

Feel free to ask me to look over your solutions, or to direct you to relevant sections in your old textbooks.

However, I will not give solutions to any questions.